EVOLUTION OF A 1.25 $\mathcal{M}_\odot$ STAR

A Journey Inspired by Astrophysicist Dr. Carl Sagan

A close-up photo of the bright center of a star cluster, Photo courtesy of NASA (https://unsplash.com/photos/OVO8nK-7Rfs)

By Michelle Babcock

"The Cosmos is rich beyond measure: the total number of stars in the universe is greater than all the grains of sand on all the beaches of the planet Earth." —Astrophysicist Dr. Carl Sagan

INTRODUCTION

Look up at the sky on a clear night, and you may see dozens or hundreds of stars with the naked eye, depending on the amount of ambient light in your area. If you're lucky enough to visit a dark sky area with little or no light pollution at some point in your life, you may see thousands of stars, along with our Milky Way galaxy.

From planet Earth, our edge-on view of the Milky Way galaxy appears as if a giant paint brush of light were dragged from one horizon to the other. This streak of milky light in our dark sky is made up of hundreds of billions of stars — somewhere between 100 and 400 billion — an unimaginable number.

Each star, in a cosmic sense, is born, lives, and dies, over a timescale of billions of years. Around those stars, some 160 billion planets orbit other suns. In such a vastness of space and immensity of time, it can be difficult to grasp how these massive objects with astronomical timelines relate to us humans, who live for a relatively short time on one small planet, orbiting an average star.

"We are star stuff harvesting sunlight." —Sagan

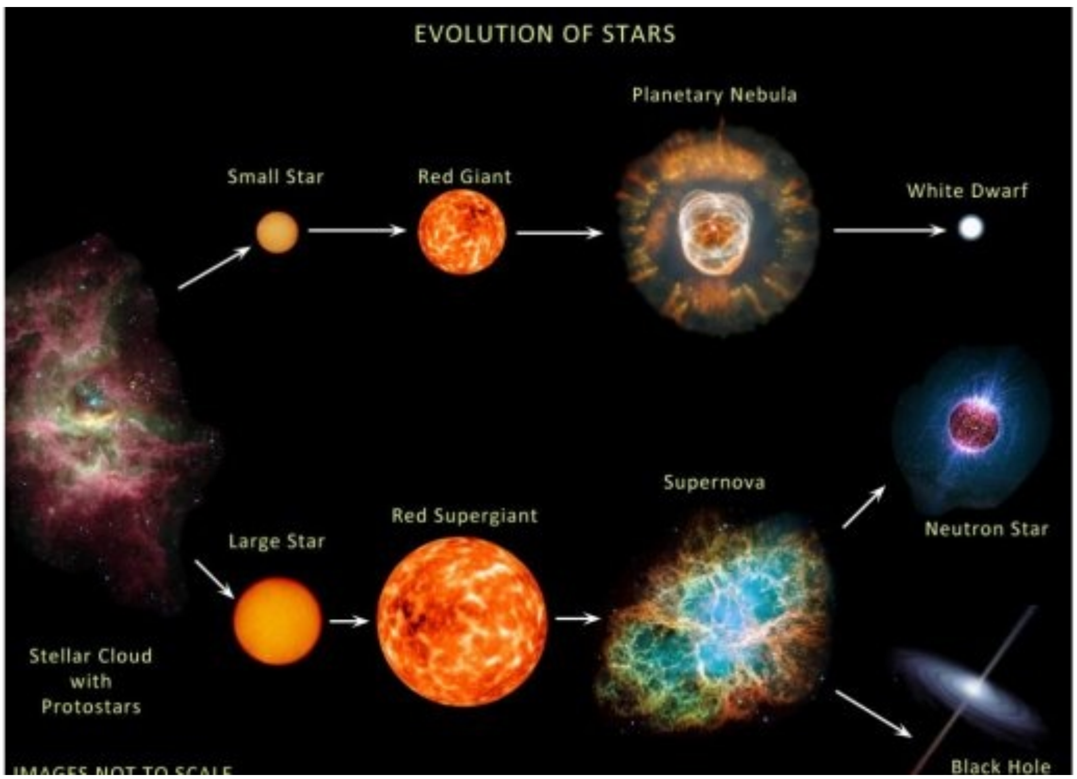
On closer inspection, it's easy to find how we are profoundly connected to the stars in more ways than one. We depend on the sun for light, for energy, for life. Without it, all plant and animal life on our planet would cease to exist. It gives us warmth, and is crucial to the environment in which our planet's plants and animals thrive. Not only is our star one of the reasons life here on earth is so hospitable, we are also connected chemically to the story of stars.

"The nitrogen in our DNA, the calcium in our teeth, the iron in our blood, the carbon in our apple pies were made in the interiors of collapsing stars. We are made of starstuff." —Sagan

Our sun is a precious commodity to life on earth, and because of this and its close proximity to us, it's the star we know most about. In fact, when astronomers study other stars in our galaxy and universe, they use our sun as a way to measure the mass of other stars: this is called "solar mass". Our sun is one solar mass, or "1.00$\mathscr{M}_{\odot}$".

In the following pages, we will explore the evolution of a 1.25$\mathscr{M}_{\odot}$ star, at some points comparing it to a 1.00$\mathscr{M}_{\odot}$ star similar to the sun. We will follow the journey of this $1.25\mathscr{M}\odot$ star from what astronomers liken to its conception and birth in stellar nurseries called nebulae into the protostar stages, all the way to its death.

Depending on the initial mass of a star, whether it's large or small, it may die in one of a few ways. Low mass stars, between $0.5\mathscr{M}\odot$ and $8\mathscr{M}\odot$, end up ejecting a planetary nebula and leaving behind their naked, dense core, called a white dwarf. High mass stars, with greater than $8\mathscr{M}\odot$, will experience an astronomically violent explosion called a supernova, and will end up as either a neutron star or black hole.



Evolution of Stars, Courtesy of ROOTS Magazine (https://www.casopisroots.cz/pozorovani-nocni-oblohy-duben/)

Our $1.25\mathscr{M}\odot$ star will end its life as a white dwarf after ejecting a planetary nebula, much like the eventual death of our own $1.00\mathscr{M}\odot$ sun. In the following pages, we will follow the path of this star's evolution, gaining an intimate understanding of its birth, life, and death. We'll use our imagination to visualize each stage, alongside physics and mathematics to present an accurate analysis of such stages.

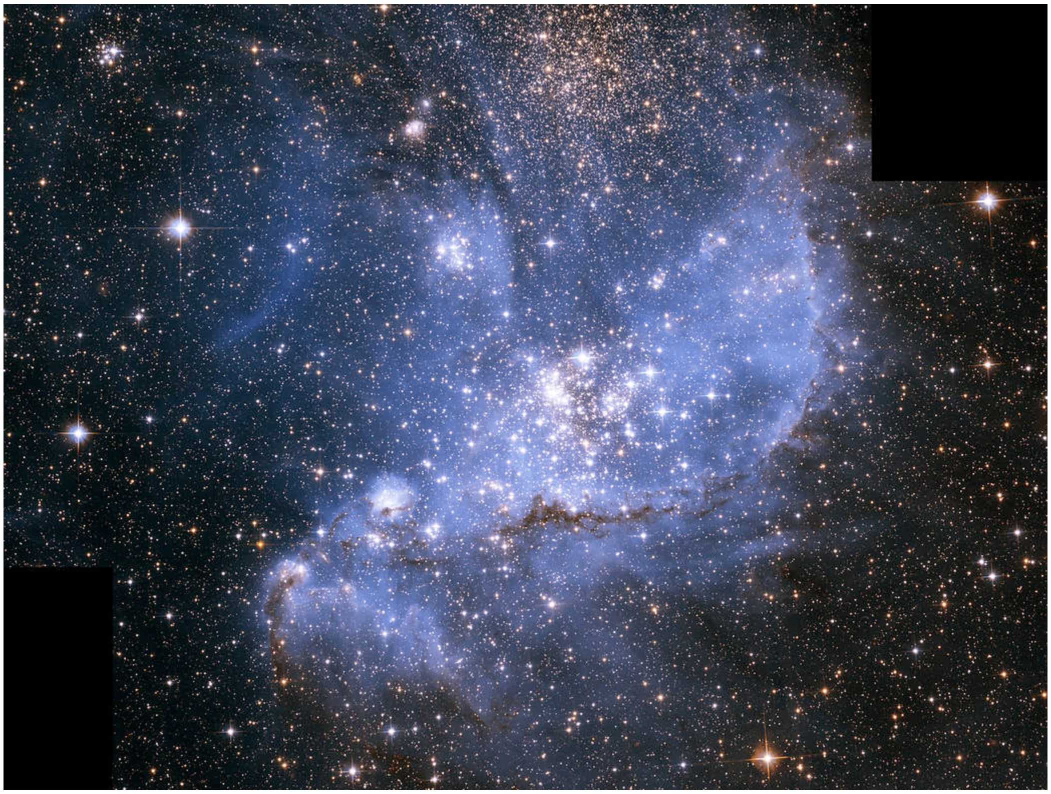
A STAR IS BORN

Within the Inter-Stellar Medium (ISM), the space between stars, there are areas of gas and dust called nebulae. These nebulae are known as stellar nurseries, and can produce hundreds or thousands of stars in areas of a nebula where the conditions are right.

Nebulae can vary in temperature, from about 10 to 10,000 K, and in density, between about 1,000 to 10,000 hydrogen atoms/cm^3. For perspective, the air we breathe is about 10 quintillion molecules/cm^3. There are several classes of nebulae, depending on their contents.

The most massive objects in our galaxy are giant molecular clouds, which are a type of nebula where there is both gas and dust. These giant clouds typically have a mass of 100 to 1,000,000 times the mass of our sun, and can be about 15 to 60 parsecs (pc) wide, equal to about 49-196 light-years or 288-1,152 trillion miles wide.

Inside giant molecular clouds, temperatures are typically at around 10 degrees Kelvin, equivalent to about -441.67 degrees Fahrenheit. The colder a nebula is, the slower molecules move and thus the more tightly together those molecules are packed: density increases as temperature decreases.



2005 Image of nebula 346 shows infant stars forming from gravitational collapse, which have not yet begun fusion.
https://www.nasa.gov/image-feature/goddard/2018/hubble-exquisite-view-of-a-stellar-nursery

With the extremely low temperatures present in Giant Molecular Clouds, the density of hydrogen in certain areas of the nebula becomes enough that gravitational collapse can occur with small concentrations of mass. Gravitational collapse happens when there is sufficient gravitational attraction and weak internal pressure in an cloud-like area of the nebula.

According to the Viral Theorem and Jean's Instability, if two times the internal pressure is greater than the gravitational attraction, then the internal gas pressure does not allow for collapse. However, if two times the internal gas pressure is less than the force of gravity, then self gravity dominates the system, causing a collapse.

$$2K + U = 0$$

where $U \sim -3/5\ G\mathcal{M}^2_c / R_c$ and $K = 3/2\ NkT$ and $N = \mathcal{M}_c/\mu m_H$

We can determine the minimum mass needed for a collapse to occur by using the equation for Jean's Mass, which is about 0.8 times the mass of the sun, or 80 times the mass of planet Jupiter. For collapse to occur, the mass of the cloud ($\mathcal{M}_c$) must be greater than Jean's Mass ($\mathcal{M}_J$).

$$\mathcal{M}_c > \mathcal{M}_J$$

$$\mathcal{M}_J = (5kT/G\mu m_H)^{3/2}\ (3/(4\pi\rho_o))^{1/2}$$

$$\mathcal{M}_c = (5kT/G\mu m_H)\ (3\mathcal{M}_c\ /4\ \pi\rho_o)^{1/3}$$

Based on spectral analysis, we know Jupiter is made up of mostly hydrogen (about 90%) and helium (about 10%), the same elements we find in stars. But Jupiter's mass was not big enough to cause a collapse to happen, and that is why we have a single star in our planetary system, rather than two. Similarly, if Saturn, another gaseous planet (about 94% hydrogen, 6% helium), had a mass large enough to cause collapse, then it too, might have become another star in our planetary solar system.

"Had Jupiter been several dozen times more massive, the matter in its interior would have undergone thermonuclear reactions, and Jupiter would have begun to shine by its own light. The largest planet is a star that failed." —Sagan

The period of rapid collapse happens quickly, usually taking only a few thousand years — a short time relative to the star's several billion year lifetime. During collapse, gravitational energy is released, causing the temperature to increase rapidly during the same time as the radius is shrinking rapidly.

Internal pressure builds, and that pressure begins to compete with gravity. At a certain point, the area of gaseous cloud undergoing collapse reaches the Hayashi Limit, when the internal pressure and force of gravity attain relative balance called hydrostatic equilibrium, becoming more like a star and less like a cloud. At this point, it becomes a protostar. Hydrostatic equilibrium is governed by the equation:

$$(dP/dr) = -(GM_r\rho)/r^2$$

For the next several million years, the convective protostar will slowly contract as small amounts of energy escape from the protostar in the form of heat. The protostar will continue to become hotter and smaller, until pressure and temperature become high enough for thermonuclear burning to begin. As soon as Hydrogen begins to fuse into helium in the core, the star is born.

It would take a star the size of our sun about 10 million years to reach this point. For larger mass stars, this would take less time, and for smaller mass stars it would take longer. After the star has begun fusion in its core, a brief and unexplained period called the “T Tauri phase” happens, where any remaining gas and dust cloud is blown away by solar wind (up to 75% of the original protostar cloud).

MAIN SEQUENCE

Next, the star will experience a long period of consistent luminosity and stability as it enters what is called the main sequence part of its lifetime. The main sequence represents the longest stage in stellar evolution, and a star similar in mass to our sun will be in its main sequence for about 10 billion years.

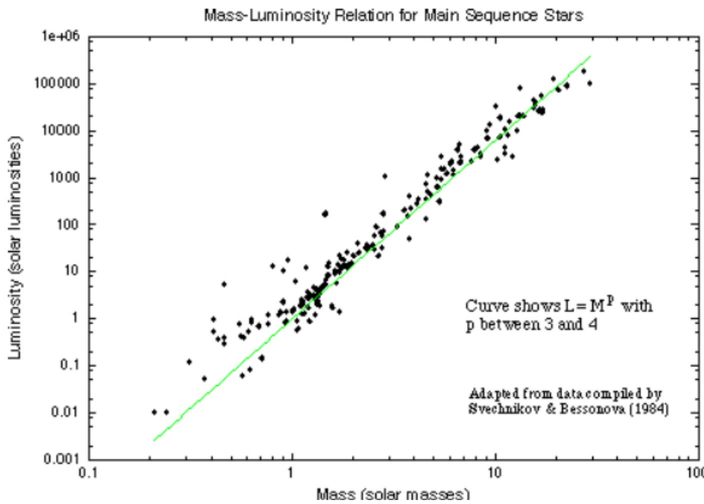
"The size and age of the Cosmos are beyond ordinary human understanding. Lost somewhere between immensity and eternity is our tiny planetary home." —Sagan

The mass ($\mathscr{M}$) of a star is directly related to its lifetime (τ) on the main sequence, as well as its luminosity, according to two relations. The larger the mass of a star, the shorter its lifetime will be, and the brighter its luminosity will be.

$$\tau \propto \mathscr{M}^{-2.5} \qquad\qquad L \propto \mathscr{M}^{3.5 \text{ to } 4}$$

The following chart shows the mass-luminosity relation for main sequence stars; as mass increases, luminosity increases. Energy conservation dictates that the rate of energy created is equal to the rate of energy output, or luminosity:

$$(dL_r/dr) = 4\pi r^2 \rho\varepsilon$$



The mass-luminosity relation for 192 stars in double-lined spectroscopic binary systems (https://www.astronomynotes.com/starsun/s8.htm)

A star's lifetime (τ) on the main sequence can be determined using a simple equation. For our $1.25\mathscr{M}\odot$ star, its lifetime on the main sequence will be approximately 5.12 billion years.

$$\tau = (1/\mathscr{M}\odot^{3})\ (10 \text{ x } 10^{9}) \text{ years}$$

$$\tau = (1/1.25^{3})\ (10 \text{ x } 10^{9}) \text{ years}$$

$$\tau = (5.12)\ (10 \text{ x } 10^{9}) \text{ years} = 5.12 \text{ billion years}$$

Evolution on the main sequence takes a long time, as the star slowly fuses Hydrogen into Helium, and eventually into heavier elements, in its core. As matter is converted into energy through fusion reactions and energy escapes from the star's core, luminosity is produced.

"The lifetime of a human being is measured by decades, the lifetime of the Sun is a hundred million times longer. Compared to a star, we are like mayflies, fleeting ephemeral creatures who live out their lives in the course of a single day." —Sagan

Stellar Surface

The radius (R), luminosity (L), and temperature (T) of a star are related according to the Stefan-Boltzmann Law, which can be seen in our data for the 1.25$\mathscr{M}_\odot$ and 1.00$\mathscr{M}_\odot$ stars. The Stefan-Boltzmann Law and relation between luminosity, radius, and temperature, are as follows:

$$L = 4\pi R^2 \sigma T^4$$

$$L \propto R^2 T^4$$

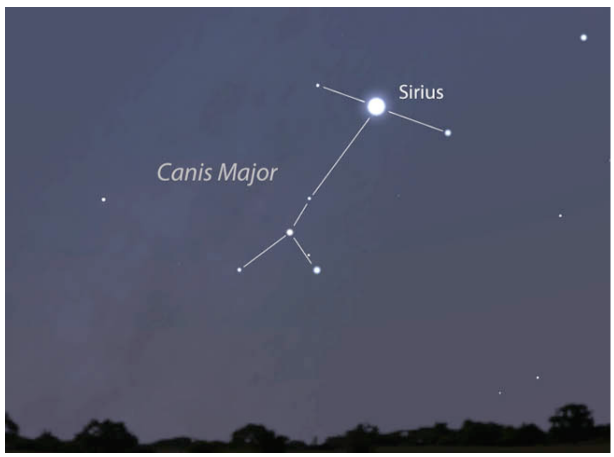
Using the radius, luminosity, and temperature ratios of our 1.25$\mathscr{M}_\odot$ and the 1.00$\mathscr{M}_\odot$ stars, we can observe the Stefan-Boltzmann Law in action. Our 1.25$\mathscr{M}_\odot$ star has both a larger radius, and a hotter temperature, compared to the 1.00$\mathscr{M}_\odot$ star. As expected, our star is also more luminous.

Surface Conditions	1.25$\mathscr{M}_\odot$	1.00$\mathscr{M}_\odot$	1.25$\mathscr{M}_\odot$/1.00$\mathscr{M}_\odot$
Radius ($R_\odot$)	1.052986	1.020998	1.031330
Luminosity ($L_\odot$)	2.69362	0.86071	3.12953
Temperature (K)	7203.6	5500.2	1.30969

With the surface conditions of our 1.25$\mathscr{M}_\odot$ star, we can also calculate several important pieces of information about our star with these given values, including the absolute magnitude of our 1.25$\mathscr{M}_\odot$ star, its lifetime on the main sequence, and approximately which spectral type it is.

Absolute Magnitude

The apparent magnitude of a star (or other celestial object such as nebulae and star clusters) tells us how bright it appears from earth, while absolute magnitude is a way for astronomers to compare the intrinsic brightness of these objects. An easy way to understand this is to think of apparent magnitude as how bright a star appears from our vantage point on earth, while absolute magnitude represents a star's brightness if all stars were viewed at the same distance from earth.



Sirius is the brightest star in our night sky, with an apparent magnitude of -1.44
https://www.universetoday.com/110605/sirius-ufo-trickster-extraordinaire/

For example, if you're standing right next to a match as it burns, it might appear brighter than a distant star. However, if you move the match flame to the same distance away as that distant star, it's obvious that the star is much brighter than the flame. To analyze stars and other celestial objects, we use a formula to determine their absolute magnitude (M), which can be found once we know the apparent magnitude (m) and distance (d) from earth.

$$m - M = 5 \log_{10} (d/10)$$

This intrinsic brightness, or absolute magnitude, is an important tool in astronomy for comparing stars and other celestial objects, regardless of how they might appear from earth.

The absolute magnitude ($M_{1.25\odot}$) of our 1.25$\mathscr{M}\odot$ star is given by the following equation, using the luminosities (L) of the 1.00$\mathscr{M}\odot$ star and our star, and the absolute magnitude of the 1.00$\mathscr{M}\odot$ star ($M_{1.00\odot}$):

$$M_{1.00\odot} - M_{1.25\odot} = 2.5 \log_{10} (L_{1.25\odot}/L_{1.00\odot})$$

$$M_{1.25\odot} = 4.75 - 2.5 \log_{10} (2.69362/0.86071)$$

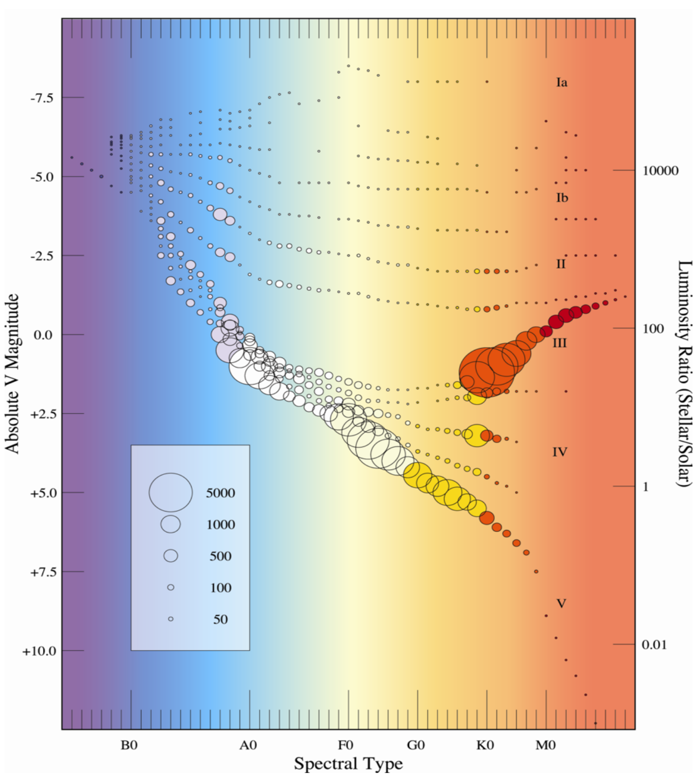
$$M_{1.25\odot} = 3.51$$

When comparing absolute magnitudes, it's important to note how the scale works. The more negative, or the smaller the number, the brighter the object. The more positive the number, the dimmer the object. The following chart shows the apparent and absolute magnitudes of several stars, in order of the brightest absolute magnitude, to the least bright absolute magnitude.

Star	Apparent	Absolute
Rigel	0.14	-7.1
Betelgeus	0.45	-5.14
Vega	0.03	0.58
Sirius	-1.44	1.41
The Sun	-26.93	4.83
GJ 75	5.63	5.63

As we can see from the above chart, Vega, Sirius, and the Sun appear to be the brightest stars in this chart when viewed from earth (apparent magnitude). However, when we when compare these to stars like Rigel and Betelgeus using the formula to find absolute magnitude, we find that some stars with a dimmer apparent magnitude are actually intrinsically brighter. Similarly, the opposite can be true; some stars that appear brighter to us on earth, have a much dimmer absolute magnitude. Or, like in the case of the star GJ 75, sometimes apparent and absolute magnitude are the same or very close, if the star is located about 10 pc from earth.

"There was a time before television, before motion pictures, before radio, before books. The greatest part of human existence was spent in such a time. Over the dying embers of the campfire, on a moonless night, we watched the stars." —Sagan



Hertzsprung-Russell Diagram
Sowell et al, Astronomical Journal, 134, 1089, 2007

Combining the absolute magnitude, luminosity ratio, and surface temperature, this Hertzsprung-Russell Diagram (previous image) gives us a better understanding of the abundance of different spectral types of stars, and how their absolute magnitude, luminosity ratio, and surface temperature (decreasing in temperature from left to right) relate to their spectral classification.

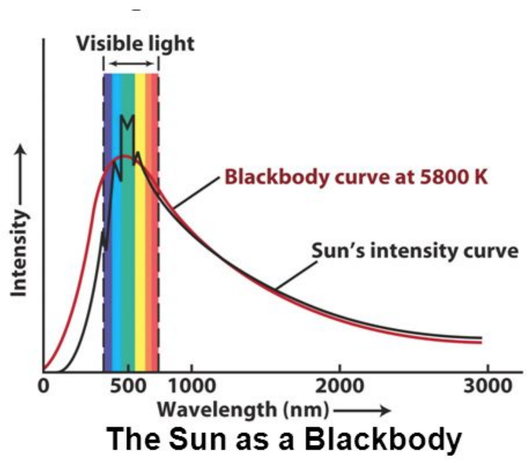
Spectral Type

Stars are classified based on their mass, luminosity, radius, and surface temperature, with O representing the hottest stars with surface temperatures higher than 25,000 K, and M representing the coolest stars with surface temperatures below 3,500 K. We don't always know a star's surface temperature, but astronomers can use spectroscopy and the idea of blackbody radiation to determine a star's approximate temperature and spectral type.

A blackbody is an idealized object that absorbs all energy that touches it, and heats up until it emits energy at the same rate at which it's absorbed. Therefore, it emits energy at all wavelengths, and the hotter the temperature, the more energy it emits. In addition, the wavelength at which maximum energy is emitted will be shorter, the hotter the temperature.

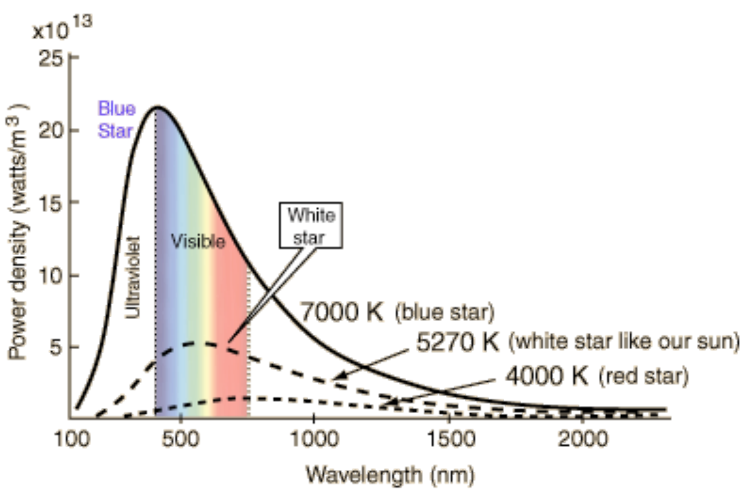
"Across the sea of space, the stars are other suns." —Sagan

If we compare the sun's electromagnetic radiation curve with that of an idealized blackbody at approximately the same temperature as the sun, we can see that blackbodies are a great approximation for stars.



The Nature of Light, https://slideplayer.com/slide/8774749/

Since the maximum wavelength of a blackbody varies with temperature, astronomers can use blackbody curves at various temperatures and spectroscopy to determine the surface temperatures of distant stars and use that information to help classify each star's spectral type.



Star Temperatures, https://ef.engr.utk.edu/hyperphysics/hbase/wien.html

Using Wien's Law, we can use the maximum blackbody wavelength to determine surface temperature, or use the surface temperature to determine the maximum wavelength.

$$\lambda_{max} = 0.0029/T$$

For our 1.25$\mathscr{M}_{\odot}$ star: $\lambda_{max} = 0.0029/7203.6$

$$\lambda_{max} = 403 \text{ nm}$$

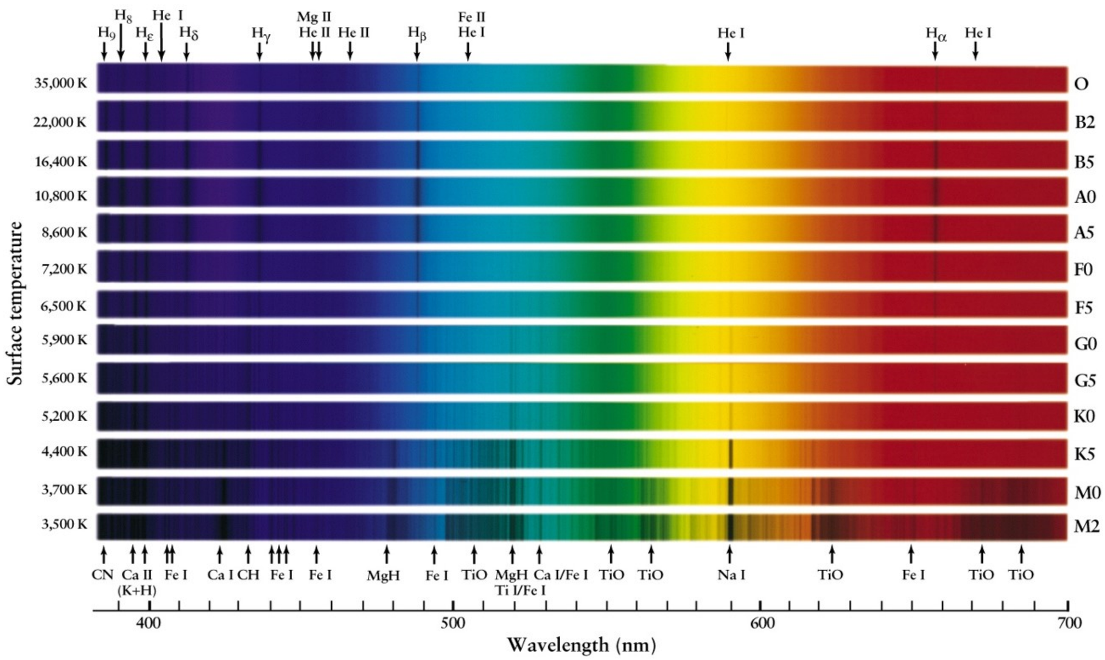
Our star's maximum wavelength is 403 nm, which is consistent with what we would expect from a star with a temperature of 7203.6 K.

Class	Color	Prominent Spectral Lines	Surface Temp. (K)
O	Blue	Ionized helium, hydrogen	> 25,000 K
B	Blue-white	Neutral helium, hydrogen	11,000 – 25,000 K
A	White	Hydrogen, ionized sodium and calcium	7,500 – 11,000 K
F	White	Hydrogen, ionized and neutral sodium and calcium	6,000 – 7,500 K
G	Yellow	Neutral sodium and calcium, ionized calcium, iron, magnesium	5,000 – 6,000 K
K	Orange	Neutral calcium, iron, magnesium	3,500 – 5,000 K
M	Red	Neutral iron, magnesium, and neutral titanium oxide	< 3,500 K

Spectral Types, https://freeburgkm.wordpress.com/2014/12/04/thermal-radiation/

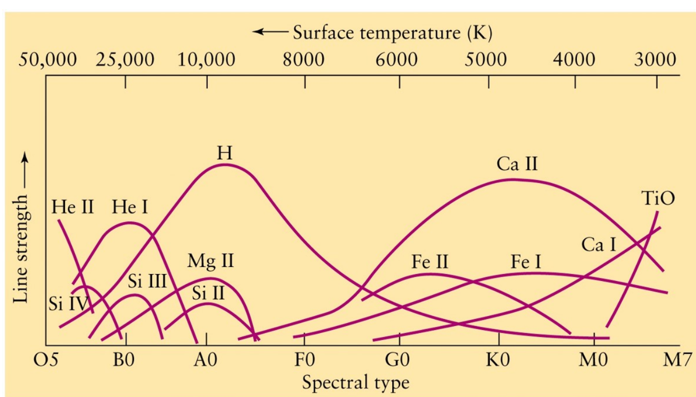
Going back to spectral classification, a star is classified as one of seven spectral types — O, B, A, F, G, K, M — based on its surface temperature. In addition to the main spectral class, each star can be assigned a subtype from 0 to 9, with 0 representing the hottest and 9 representing the coolest stars within each class. For example, our sun is a G2 star, with a surface temperature of about 5,700 K.

Using spectroscopy, we can see that each class of stars has a unique spectral thumbprint. The following graphic shows the different spectral lines from each temperature and spectral type.



Universe, Kaufmann, W.H.Freeman and Company

For stars with a similar composition to the sun, we can use the following chart to determine which spectral lines would be the most prevalent.



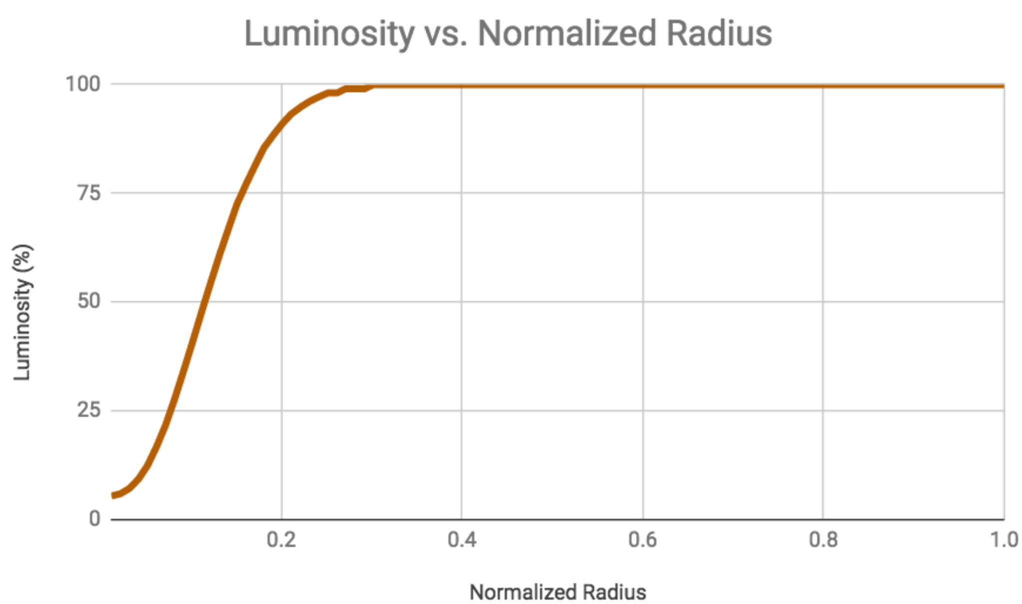
Universe, Kaufmann, W.H.Freeman and Company

Based on the surface temperature of our $1.25\mathcal{M}_{\odot}$ star, at 7203.6 K, it would be an F0 spectral type, and it will appear yellow-white. As an F0 spectral type, our $1.25\mathcal{M}_{\odot}$ star would have a medium strong hydrogen (H) spectral line, as well as lines for singly ionized calcium (Ca II), and iron (Fe I).

If we were to use our star's mass to calculate spectral type, it would be an F6, with slightly less strong hydrogen (H) line, a slightly stronger calcium (Ca II) line, and might also have a line for singly ionized iron (Fe II).

Inside the Core

Taking a closer look at data provided for our 1.25$\mathscr{M}_\odot$ star, we can plot luminosity, temperature, pressure, and mass, as a function of the normalized radius of our star. Beginning with the plot for luminosity vs. normalized radius.

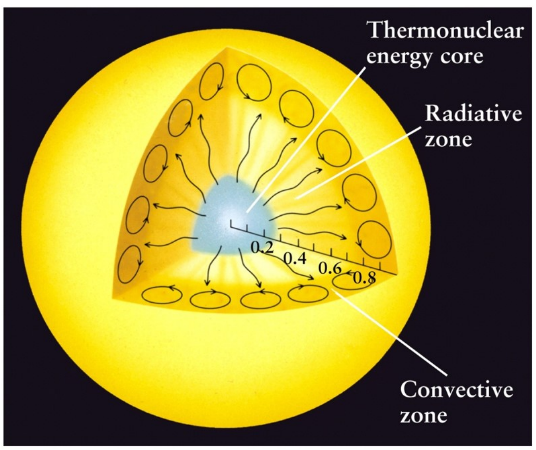


This luminosity vs. normalized radius plot shows the luminosity depending on how close we are to the star's center. Knowing that inside the core fusion is occuring, and fusion takes very high temperature and pressure to take place, the core region will be producing nearly all of the energy and thus luminosity of the star.

Using this information, we can determine at what point along the normalized radius 99% of the luminosity is produced, and in doing so we can define this are the star's core region. To do this, we can determine what 99% of the total luminosity (L) is for our 1.25$\mathscr{M}_\odot$ star, then trace that luminosity back to the corresponding radius.

$$L\ (0.99) = 1.03 \times 10^{34}\ (0.99) = 1.02 \times 10^{34}$$

At approximately 0.282 (28.2%) of the normalized radius, which is equal to about 2.07 x 10^{10} cm or about 207,000 km, 99% of the luminosity is produced and can be defined as the core of our star. Inside the core, energy is created through thermonuclear reactions (fusion) and is radiated outward. Once in the star's envelope, outside the core area of the star, energy transport is convective. Radiative energy transport is governed by the equation: $(dT/dr) = -(3/4ac)(k\rho/T^3)(L_r/4\pi r^2)$



Universe, Kaufmann, W.H.Freeman and Comp

From our calculated core radius, we can determine the core volume is 2.25% of the total volume of our 1.25$\mathscr{M}_\odot$ star, using the formula for the volume (V) of a sphere. We can also see the total volume is more than 44 times larger than the core.

$$V = (4/3)\ \pi\ R^3$$

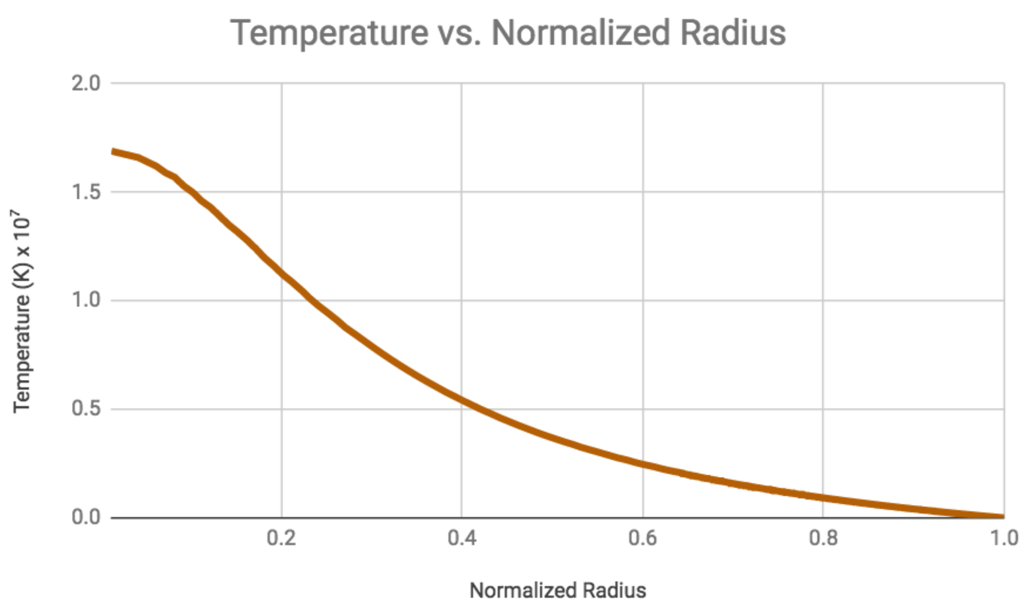
Core volume: V = (4/3) π (2.07 cm)^3 = 37.15 cm^3

Total volume: V = (4/3) π (7.33 cm)^3 = 1,647.68 cm^3

Core volume/Total volume = 37.15/1,647.68 = 0.02255 x 100 = 2.25%

Total/Core = 1,647.68/37.15 = total is 44.35 x core

Looking at a plot of temperature vs. normalized radius, we can see the impacts of energy transport from the core.



As expected, inside the core we see the highest temperatures, reaching 16,900,000 K, or 1.69 x 10^7 K, where fusion is occuring. As the energy radiates outward from the core, it gradually lowers in temperature as it moves away from the core and is spread over a greater area. Once in the star's envelope, the energy transport is convective and continues to drop in temperature as expected.

One set of fusion reactions that occurs during the main sequence of a star's evolution as hydrogen fuses into helium is called the proton-proton (PP) chain reaction. In the PPI chain reaction, the most common PP reaction, what begin as four protons (^{1}H), fuse into deuterium (^{2}H), then into a helium isotope (^{3}He), and eventually into a helium particle (^{4}He) with two protons and neutrons. In the process, a positron (e^{+}), neutrino (ν), and photon (γ) are released.

$$^{1}H + {}^{1}H \rightarrow {}^{2}H + e^{+} + \nu$$

$$^{1}H + {}^{2}H \rightarrow {}^{3}He + \gamma$$

$$^{3}He + {}^{3}He \rightarrow {}^{4}He + {}^{1}H + {}^{1}H$$

Since the initial mass of the four protons is more than the final mass of the particles produced in this chain reaction, the difference in mass is converted into energy through Einstein's equation, $E = mc^2$. The PP cycle is responsible for the majority of energy produced in lower mass stars with temperatures below about 18 million Kelvin.

"Atoms are made in the insides of stars. In most of the stars we see, hydrogen nuclei are being jammed together to form helium nuclei. Every time a nucleus of helium is made, a photon of light is generated. This is why the stars shine." —Sagan

Another chain reaction that can occur is the carbon, nitrogen, and oxygen, or CNO cycle, which requires much hotter temperatures for the heavier elements to fuse.

$$^{12}C + {}^{1}H \rightarrow {}^{13}N + \gamma$$

$$^{13}N \rightarrow {}^{13}C + e^{+} + \nu$$

$$^{13}C + {}^{1}H \rightarrow {}^{14}N + \gamma$$

$$^{14}N + {}^{1}H \rightarrow {}^{15}O + \gamma$$

$$^{15}O \rightarrow {}^{15}N + e^{+} + \nu$$

$$^{15}N + {}^{1}H \rightarrow {}^{12}C + {}^{4}He$$

The PP and CNO cycles compete with each other to use hydrogen in fusion reactions. Stars with a core temperature hotter than about 18 million K (1.8×10^7 K), the CNO cycle is favored. Using the following formulas for energy generation rates (ε), we can calculate the energy released per gram per second for both the PP and CNO chain reactions.

$$\varepsilon_{PP} = C_{PP}\, \boldsymbol{\rho}_{core}\, X^2\, (10^6/T_{core})^{2/3}\, e\hat{}(-33.8(10^6/T_{core})^{1/3}$$

$$\varepsilon_{CNO} = C_{CNO}\, \boldsymbol{\rho}_{core}\, XX_{CNO}\, (10^6/T_{core})^{2/3}\, e\hat{}(-152.3(10^6/T_{core})^{1/3}$$

$$\varepsilon_{PP} = (2.5 \times 10^6)\,(85.0341)(0.7)^2(10^6/1.69 \times 10^7)^{2/3}\, e\hat{}(-33.8(10^6/1.69 \times 10^7)^{1/3}$$

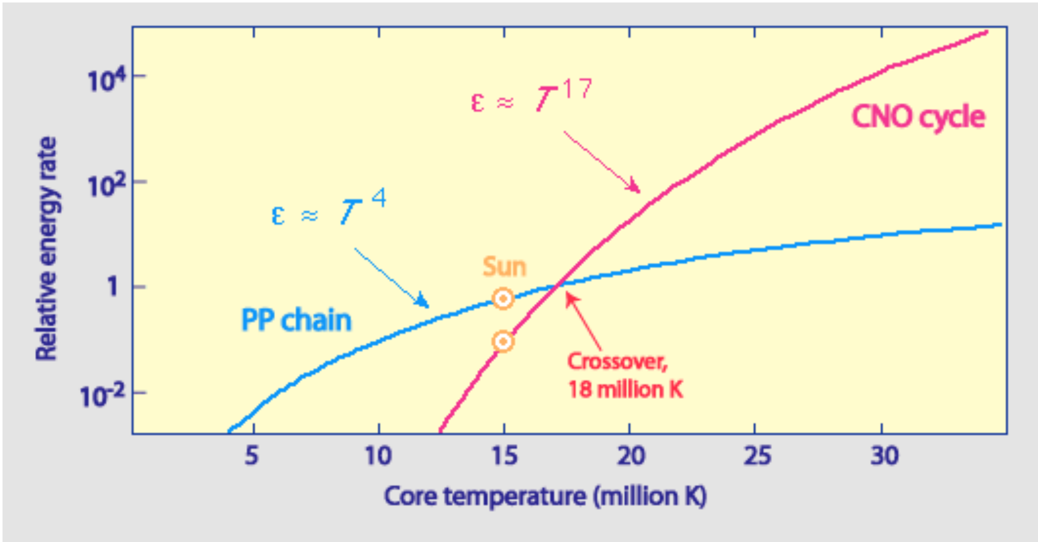
$$\varepsilon_{CNO} = (9.5 \times 10^{28})(85.0341)(0.7)(2.67 \times 10^{-3})(10^6/1.69 \times 10^7)^{2/3}\, e\hat{}(-152.3(10^6/1.69 \times 10^7)^{1/3}$$

$$\varepsilon_{PP} = 30.13104 \text{ erg/s/g} \qquad \varepsilon_{CNO} = 38.54162 \text{ erg/s/g}$$

$$(\varepsilon_{CNO}/\varepsilon_{PP}) = 38.54162/30.13104 = 1.27913 \text{ or about } 1.28$$

Based on our 1.25$\mathscr{M}_\odot$ star's core temperature (T_{core}), core density ($\boldsymbol{\rho}_{core}$), hydrogen abundance (X), and other metals abundance (Z), the CNO chain produces 38.54 erg/s/g, while the PP chain produces 30.13 erg/s/g. Thus the CNO cycle produces 1.28 times the energy of the PP cycle, released per second, per gram.

Because the CNO cycle requires hotter temperatures (above 18 million Kelvin) to dominate energy production, our 1.25$\mathscr{M}_\odot$ star's core temperature of 16.9 million Kelvin means the PP cycle dominates energy production, as seen on the graph below.



Adapted from an image by Mike Guidry, University of Tennessee, http://www.atnf.csiro.au /outreach/education/senior/astrophysics/stellarevolution_mainsequence.html

While our 1.25$\mathscr{M}\odot$ star produces energy both using the PP and CNO chains, let's assume 100% of the surface luminosity is generated by the PPI chain and calculate the number of reactions per second our star would need to produce its luminosity.

$$L_{surface} = (2.69362 \odot)(1 \odot) = 1.03 \times 10^{27} \text{ J/s}$$

$$E_{PPI} = 26.75 \text{ MeV per PPI reaction} = 4.29 \times 10^{-12} \text{ J}$$

$$E_{total} = L/t = 1.03 \times 10^{27} \text{ J} / 1 \text{ s} \qquad N_{reactions\ per\ second} = E_{total} / E_{PPI}$$

$$1.03 \times 10^{27} \text{ J} / 4.29 \times 10^{-12} \text{ J} = 2.40 \times 10^{38} \text{ reactions per second}$$

The amount of hydrogen involved in 2.40 x 10^{38} reactions per second by the PPI chain can be found with the atomic weight of hydrogen (1.007825u = 938.8 MeV) converted to Joules per atom, which is equal to 1.504 x 10^{-10}. With this, we can use the luminosity of our star in Joules per second to calculate the amount of hydrogen converted to energy per second:

$$(E_{total} / 1.504 \times 10^{-10}) \ (1 \text{ second}) = N_{atoms}$$

$$(1.03 \times 10^{27} \text{ J} / 1.504 \times 10^{-10} \text{ J}) = 6.85 \times 10^{36} \text{ atoms}$$

$$1 \text{ hydrogen atom} = 1.674 \times 10^{-27} \text{ kg}$$

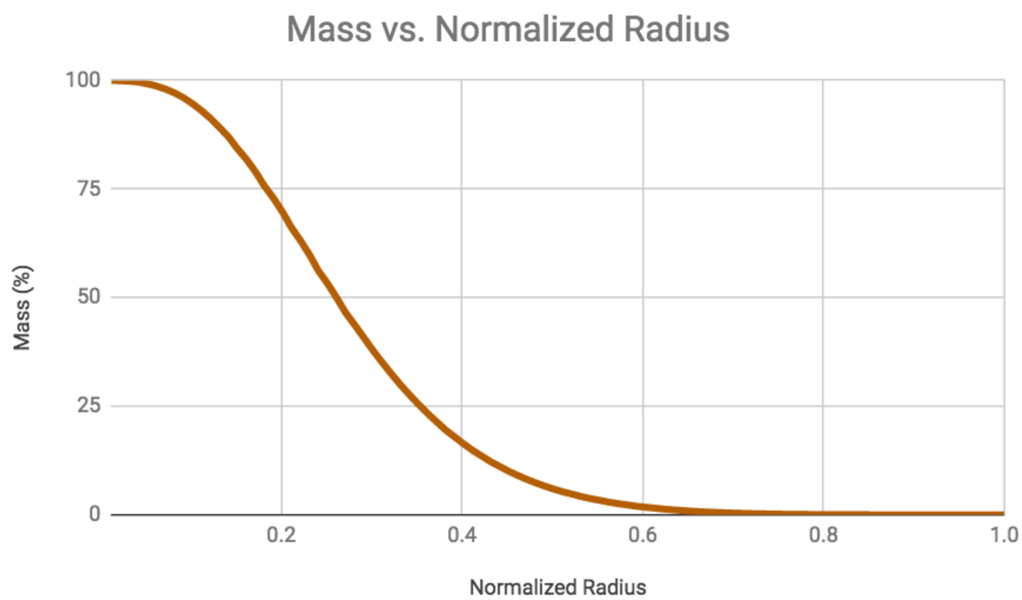
$$(1.674 \times 10^{-27} \text{ kg}) \ (6.85 \times 10^{36}) = 1.15 \times 10^{10} \text{ kg hydrogen per second}$$

Therefore, if our $1.25\mathscr{M}_\odot$ star's surface luminosity were 100% due to PPI chain reactions, there would be 2.40×10^{38} reactions per second, and each second, about 1.15×10^{10} kg of hydrogen would be converted to energy. This vast number of reactions per second is an example of how much fusion activity is needed to produce a star with roughly 2.5 times our sun's luminosity, if its energy production were dominated entirely by the PP cycle.

While this exercise gives us an idea of the huge number of reactions needed in a PP-dominated star, our star doesn't fit this exact model. For the most part, stars with a core temperature greater than 18 million Kelvin are dominated by the CNO chain, while those with cooler core temperature are dominated by PP chain reactions. However, stars may produce energies by both the PP and CNO chains at varying ratios, such as our star.

Mass and Pressure

Plotting our star's mass vs. its normalized radius, we see that most of the mass (66.72%) is concentrated within the star's core radius, while 33.26% of its mass is outside the core.



The total mass of our 1.25$\mathscr{M}_\odot$ star is given as 2.49 x 10^{30} kg, so to determine the mass of the core, we can multiply that by 0.6650, to get 1.66 x 10^{30} kg, or 0.8346$\mathscr{M}_\odot$. So while the core only accounts for 2.25% of our star's volume, it contains an astonishing 66.50% of its mass! The ratio of core mass/total mass is:

$$0.8346\mathscr{M}_\odot/1.25\mathscr{M}_\odot = 0.6677$$

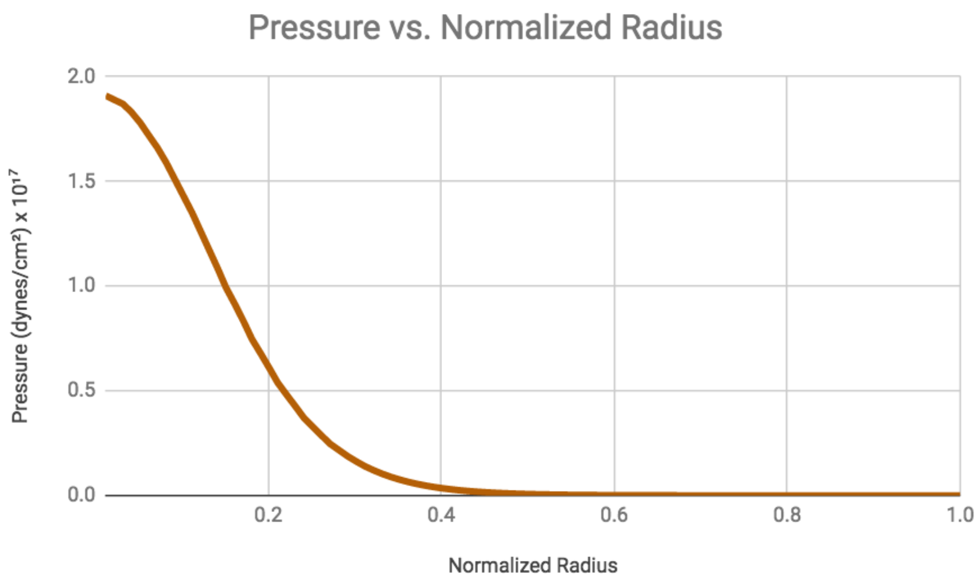
$$1.66 \times 10^{30}\ \text{kg}/2.49 \times 10^{30}\ \text{kg} = 0.6667$$

Core mass / Total mass = 0.6672 or 66.72%

The large mass of the core in such a small radius, means it's much more dense than the envelope, according to mass conservation: $(dM_r/dr) = 4\pi r^2 \rho$.

$$\rho_{core} = \text{core density} = 85.0341\ \text{g/cm}^3$$

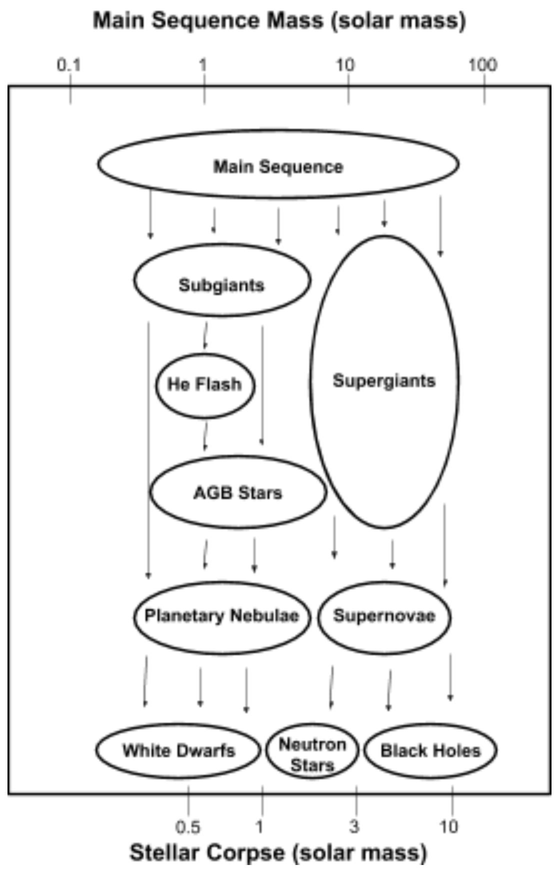
As seen in the following plot of pressure vs. normalized radius, we can see the law of mass conservation at work. Since the core is more dense than the envelope of our star, the internal pressure is much greater. Similar to the plot of mass vs. normalized radius, we can see a huge amount of pressure within the core as the mass in the core is great, and as we move away from the core, the mass density and pressure drop off. Core pressure = 1.92×10^{17} dynes/cm^2



LIFE AFTER THE MAIN SEQUENCE

"The Cosmos is all that is or was or ever will be. Our feeblest contemplations of the Cosmos stir us — there is a tingling in the spine, a catch in the voice, a faint sensation, as if a distant memory, of falling from a height. We know we are approaching the greatest of mysteries."
—Sagan

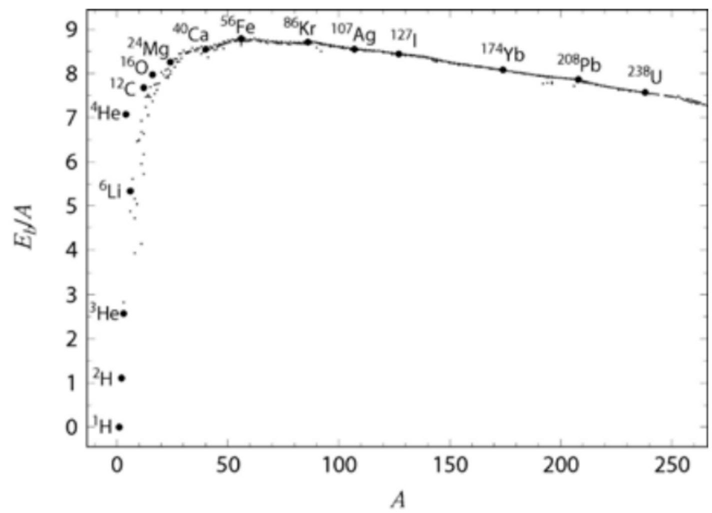
High mass stars lead an exciting life after they leave the main sequence, becoming supergiants that eventually die in a powerful and violent supernova explosion, leaving either a neutron star or black hole as their stellar corpse.



This flowchart shows the evolution of stars after they leave the main sequence, as a function of their main sequence mass, and eventual mass of their stellar corpse. Stellar Evolution graphic based on Sowell powerpoint graphic.

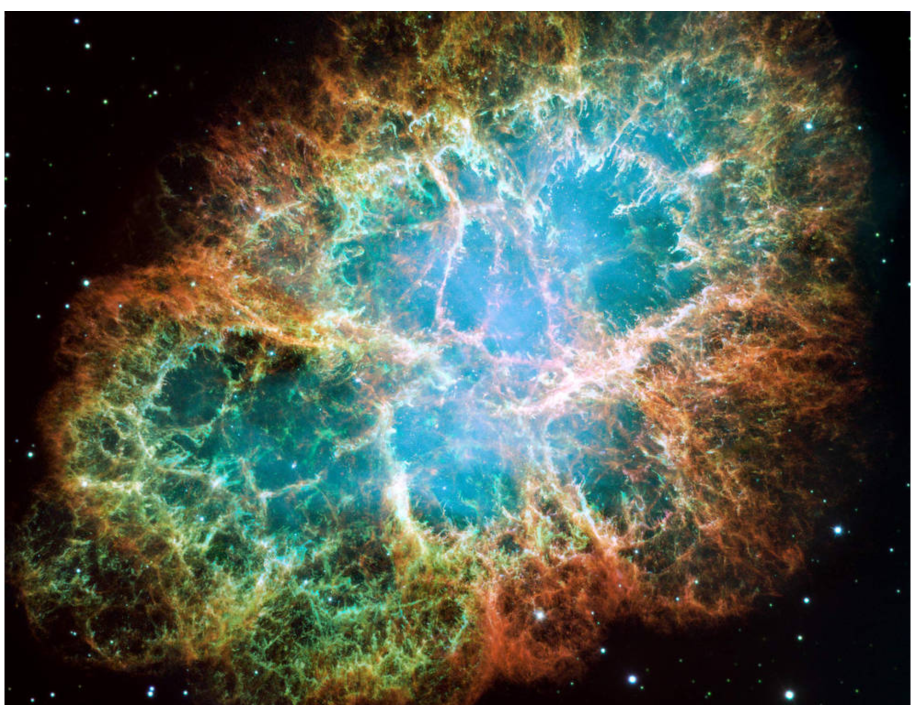
Because of its mass and temperature, after leaving the main sequence and becoming a supergiant, a star greater than about 8$\mathscr{M}_\odot$ will fuse heavier and heavier elements until iron is formed in its core.

The fusion process for all elements leading up to, and including, iron releases energy. Because of this, these elements can fuse in the star's core as stellar fuel. Once these lighter elements are all used up in fusion reactions, the star's fuel source is depleted and its core is made of heavy iron.



This Binding Energy plot shows the energy released due to the loss in mass during fusion. Image from Sowell PowerPoints

Because of this, a supergiant will end up collapsing under its own gravity, ending its life either in a magnificent explosion called a supernova, or by collapsing into a black hole.



The Crab Nebula is the remnants of a supernova explosion, NASA, ESA, J. Hester and A. Loll (Arizona State University), https://www.nasa.gov/feature/goddard/2017/messier-1-the-crab-nebula

If a star's core is less than about $3\mathscr{M}\odot$, it will end its life by collapsing in on its own gravity until the pressure of its neutron degenerate core halts the collapse, causing a supernova explosion. In supernovae, heavier elements than iron can form.

If the core is greater than $3\mathscr{M}\odot$, collapse will continue beyond the neutron degenerate core to create a black hole, with such an immense mass and gravitational pull that not even light (photons) can escape from its center.

While the deaths of high mass stars are exciting, our $1.25\mathscr{M}\odot$ will experience a very different evolution after it leaves the main sequence. After about 10 percent of the star's mass is depleted of hydrogen, it will move off of the main sequence and become a subgiant. At that point, it has burned through much of its hydrogen, leaving a non-burning helium core surrounded by a hydrogen shell. Our star will spend about a twentieth of its lifetime as a subgiant, as its radius expands and the stellar surface cools until reaching the Hayashi Limit.

The temperature and speed of helium nuclei in the core continue to increase. Eventually, the pressure and temperature in the core become too much and once again, fusion begins in the core. But, this time, the core experiences a runaway energy generation through helium burning in what's called the helium flash. This runaway helium flash only lasts for a few seconds but can be 100 billion times more luminous than the sun.

Following the helium flash, our $1.25\mathscr{M}\odot$ star enters the red giant phase. Its core will contain helium fusing into carbon, surrounded by a hydrogen shell. The red giant phase represents our star's second-longest phase, with the main sequence phase being the longest by far. Our star will be a red giant for about one tenth of the time it spent on the main sequence.

Our 1.25$\mathscr{M}_\odot$ star will experience several more changes, including a core contraction and a double-shell burning stage, known as the asymptotic giant branch as its now carbon-oxygen core continues to increase in mass. This carbon-oxygen core is a signal that the end of our star is near.

Near the end, our star might experience one or more phases of pulsational instability, which can be observed by changes in its luminosity as the star's envelope fails to reach a balance of hydrostatic equilibrium as its gravity and pressure fight against the imbalance.

NGC 2392, also known as the "Eskimo Nebula" is a planetary nebula about 4,200 light years from the earth. NASA, https://www.nasa.gov/mission_pages/chandra/images/index.html

Eventually, our star will begin to eject mass out into space, producing what's called a planetary nebula. As it ejects this mass, its non-burning electron degenerate core is exposed. This “naked” carbon-oxygen or helium core is known as a white dwarf, the stellar corpse of our once luminous star.

This white dwarf, now small in size, contains an immensity of mass, and will therefore have an enormous density. For example, a white dwarf with $1\mathscr{M}_\odot$ (similar to our $1.25\mathscr{M}_\odot$ star but slightly less massive) may be the size of planet earth, but would have the mass of the sun. If we could fill a teaspoon with white dwarf material, it would weigh several tons.

COMPARED TO BETA VIRGINIS

Now, let’s do a comparison of some key data points between our $1.25\mathscr{M}_\odot$ star and Beta Virginis, another $1.25\mathscr{M}_\odot$ star, which is visible in the Virgo constellation about 35.6 light years from the sun.

Surface Conditions	Our Star	Beta Virginis
Radius ($R_\odot$)	1.053	1.681
Luminosity ($L_\odot$)	2.69	3.80
Temperature (K)	7203	6243
Spectral Type	F0-F6	F8
Absolute Magnitude	3.51	3.41

Beta Virginis has a radius 1.6 times larger than the radius of our star, a luminosity 1.4 times our star’s luminosity, and its temperature is cooler than our star, at about 0.87 times our star’s temperature. The spectral type for Beta Virginis resembles a lower temperature spectral type than our star, which we would expect.

CONCLUSION

"Of the thousands of stars you see when you look up at the night sky every one of them is living in an interval between two collapses. An initial collapse of a dark interstellar gas cloud to form the star and a final collapse of the luminous star on the way to its ultimate fate. Gravity makes stars contract unless some other force intervenes.

The sun is an immense ball of radiating hydrogen. The hot gas in its interior tries to make the sun expand. The gravity tries to make the sun contract. The present state of the sun is the balance of these two forces an equilibrium between gravity and nuclear fire. In this long middle age between collapses the stars steadily shine."—Sagan

From stellar birth in giant molecular clouds, to the awesome deaths of stars of different mass, we've gained an intimate understanding of the evolution of a $1.25\mathscr{M}_\odot$ star, at times comparing it to other similar stars.

As we did at the beginning of this journey, let's take a moment to think about our place among these stars. We float through space on our tiny planet, orbiting a $1.00\mathscr{M}_\odot$ star, surrounded by billions of other stars. With this perspective, look anew at the night sky. Each of those stars is another sun, which lives and dies over billions of years.

"The cosmos is within us. We are made of star-stuff. We are a way for the universe to know itself." —Sagan

Compiled photographs of the progression of the Aug. 21, 2017 Total Solar Eclipse from directly under the path of totality, By Michelle Babcock.

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